Towards an Operational Interpretation of Fuzzy Measures

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Abstract

In this paper we propose an operational interpretation of general fuzzy measures. On the basis of this interpretation, we define the concept of coherence with respect to a partial information, and propose a rule of inference similar to the natural extension [3].

Keywords: Fuzzy measure, partial information, coherence, extension.

1 Introduction

Two main problems arise when applying fuzzy measures in practical applications. One is the lack of a clear understanding about the meaning of the measures; until now, no consensus has been reached about what the numbers mean. The other problem is that typically it is not possible to get a complete specification of the value of the measure for all the subsets in the domain, but for a reduced number of them.

Here we propose an operational interpretation of fuzzy measures in order to give a clear meaning to the numbers. This interpretation leads to a straightforward definition of coherence, and to a natural rule of inference that will allow us to make predictions about the value of the measure in the sets where it is unknown.

We start off proposing an operational interpretation and some examples in section 2. In section 3 we define the concept of *partial information* and *coherence*, which will be the basis of an inference rule called *extension*, analyzed in section 4. An algorithm for computing extension is given in section 5. We have implemented this algorithm to carry out some trials that are described in section 6. The paper ends with conclusions in section 7.

2 An Operational Interpretation

A fuzzy measure [5] is a mapping $\mu : 2^{\Omega} \rightarrow [0,1]$ verifying the following properties:

1. $\mu(\emptyset) = 0$ and $\mu(\Omega) = 1$.
2. $A \subseteq B \subseteq \Omega \Rightarrow \mu(A) \leq \mu(B)$.

In all of this paper we shall consider Ω to be a finite set of categories.

Fuzzy measures have been interpreted in different ways. We shall review some of these interpretations.

Wang and Klir [5] interpret a value $\mu(A)$ of a fuzzy measure, as the "importance" of A, assuming that the total space has importance 1 and the empty set has importance 0. The elements of Ω are regarded as quality factors, and the importance of a set A is obtained from the quality factors it contains. It is assumed that the effect of those factors is not necessarily additive. They illustrate this interpretation with the following example:

Example 1 *Consider the problem of evaluating a Chinese dish. The quality factors can be the taste* (T)*, smell* (S) *and appearance* (A)*. Hence,* $\Omega = \{T, S, A\}$*. The following measure of importance can be defined:* $\mu(\{T\}) = 0.7$, $\mu(\{S\}) = 0.1$, $\mu(\{A\}) = 0$, $\mu(\{T,S\}) = 0.9$, $\mu(\{T,A\}) = 0.8$, $\mu(\{S,A\}) = 0.3$, $\mu(\Omega) = 1$ *and* $\mu(\emptyset) = 0$*. This measure is not additive.*

The drawback of this interpretation is that it is necessary to establish a criterion to assign numbers to the sets when a new problem is considered. In other words, the meaning of the numbers depends on the problem, and it does not seem straightforward to extract a general rule for interpreting the numbers from here.

A very similar interpretation is considered by Grabisch, Nguyen and Walker [1]. Again, the numbers are interpreted as degrees of importance. They propose the following example:

Example 2 *Consider three subjects in a high school: Mathematics* (M)*, Physics* (P) *and Literature* (L)*. Hence,* $\Omega = \{M, P, L\}$*. The director is interested on defining a measure of importance over the subjects according to these considerations: scientific subjects are more important, students who succeed in a scientific subject usually are successful in the other one (i.e., being successful in both subjects should not be much more important than being in one of them) and finally, there is a big merit on being successful in a scientific subject and in Literature. The measure proposed for this problem is* $\mu(\{M\}) = \mu(\{P\}) = 0.45$*,* $\mu(\{L\}) = 0.3$*,* $\mu(\{M, P\}) = 0.5$*,* $\mu(\{M, L\}) = \mu(\{P, L\}) = 0.9$*,* $\mu(\Omega) = 1$ *and* $\mu(\emptyset) = 0$*.*

Against this interpretation, the following can be argued: what does it mean that $\mu(\{L\}) = 0.3$?. In other words, what does it mean that the importance of Literature is 0.3?. We do not find a satisfactory answer from this interpretation. One could think that it means that the importance of Literature is a 30% of the total importance, but again we could ask for the meaning of "total importance". Besides, it is necessary to define *ad hoc* rules to assess the values of the measure.

A different approach is proposed by Murofushi and Sugeno [2]. They consider an operational, but not general, interpretation, under which Choquet's integral is meaningful. The situation is as follows:

Example 3 *Let* Ω *be a set of workers in a workshop, and suppose they produce the same products. Suppose that a group of workers* $A \in \Omega$ *works in the most efficient way. Let* $\mu(A)$ *be the number of products made by* A *in one hour.* μ *can be considered as a measure of productivity.*

In the example above, μ can be normalized dividing by $\mu(\Omega)$, obtaining thus a fuzzy measure. Observe that in this situation, μ may be non-additive, since maybe two workers together produce more (or maybe less) than if they work separately.

This interpretation has an important feature that distinguishes it from the other ones: the meaning of the numbers is perfectly clear; furthermore, it is easy to assign values to the measure. However, the interpretation does not seem general enough to be applied to different situations. More precisely, we do not see how to apply it to the situations described in examples 1 and 2.

Much more controversial is the interpretation of a fuzzy measure as a measure of uncertainty. Some attempts have been made in this direction (see, for example, [1]). We think that this interpretation is not clear at all. This opinion is based on Walley's criteria for evaluating measures of uncertainty [4]. The first of these criteria is called *interpretation*, and it claims that *the measure should have a clear interpretation that is sufficiently definite to be used to guide assessment, to understand the conclusions of the system and use them as a basis for action, and to support the rules for combining and updating measures.* We think that none of the previous interpretations of fuzzy measures verify this important criterion.

Next we propose a new interpretation of fuzzy measures, general enough to cover the examples examined so far. More precisely, we shall see how our interpretation can be applied to examples 1, 2 and 3.

Assume an experiment whose possible outcomes are the elements in the power set of Ω, 2^Ω. Assume also that number 1.0 represents the total amount of resources available to the realization of the experiment, and that it coincides with the amount of resources consumed if the result of the experiment is the entire set Ω. In these conditions, for any $A \subseteq \Omega$, $\mu(A)$ can be regarded as the fraction of resources consumed if the result of the experiment is A. Let us illustrate it with an example.

Example 4 *Imagine there is a vehicle covering the connection between the harbor and the railway station in a city. This vehicle has four compartments: one for a car, one for a van, one for a motor-bike and another one for a bike. Assume that the gas tank of this vehicle has exactly the capacity necessary to carry the vehicle, with the four compartments busy, from the harbor to the railway station. Then we can regard this capacity to be equal to 1 unit. In this example,* $\Omega = \{c, v, m, b\}$*, where* c *stands for* car compartment busy*,* v *for* van compartment busy*,* m *for* motor-bike compartment busy *and* b *for* bike compartment busy*. Assume also that the vehicle does not start the trip unless at least one of the compartments is busy. All the possible transportation situations are then the elements in* 2^Ω*. In these conditions, for every* $A \subseteq \Omega$*,* $\mu(A)$ *can be interpreted as the proportion of gas consumed if* A *happens.*

According to this interpretation, the meaning of the numbers is perfectly clear, which facilitates the assessment of the values of the measure. In the example above, there is an objective way of measuring the resources consumed by the realization of the experiment, but this interpretation is still valid if the resources cannot be measured so objectively.

Note that a measure matching this interpretation is not necessarily additive, but it must be at least monotone. Here, the possible results of the experiment are sets of items or categories. Each of these individual items consumes a fraction of resources by itself. Then, if the result of the experiment consists of two items, we should not expect that they both together consume less than just one of them.

Whenever a problem is approached, the following items must be identified:

1. What the experiment consists of.
2. The set of all possible outcomes of the experiment (i.e., 2^Ω).
3. The total resources available to the realization of the experiment.
4. The amount of resources consumed by each $A \subseteq \Omega$.

Consider the situation in example 1. In this case, the experiment consists of observing the quality factors in the Chinese dish. The measure should model the preference of the observer with respect to the quality factors in the observed dish. The set of all quality factors is $\Omega = \{T, S, A\}$. Let us represent by 1 the total amount of money that the observer would pay for a Chinese dish showing the three quality factors together. In these conditions, we can interpret $\mu(A)$, for each $A \subseteq \Omega$, as the fraction of the total amount of money that the observer would pay if he observes that the dish has the factors in A.

Regarding example 2, the experiment consists of observing the subjects where a student has qualified. The possible outcomes of the experiment are the subsets in $\Omega = \{M, P, L\}$. Let us denote by 1 the maximum amount of money that the director would invest to employ a student that has qualified in the three subjects. Then, the values of $\mu(A)$, $A \subseteq \Omega$, given in example 2, can be interpreted as the fraction of that maximum amount of money that the director would invest to employ a student who succeeded in the subjects in A.

Now consider the situation in example 3. Here, the experiment consists of selecting groups of workers, and Ω is the set of all the workers in the workshop. Let us denote by 1 the price of the items made by all the workers in the workshop if they work during one hour in the most efficient way, this price being proportional to the number of items. Then, we can interpret $\mu(A)$ as the fraction of the maximum price corresponding to the items produced by the workers in A.

Thus, we have shown how the examples given so far, can be also approached according to our interpretation.

Note that in the first two situations, subjective evaluations of importance are modeled as *buying prices* with respect to a maximum. This fact could suggest that general fuzzy measures can be considered measures of uncertainty, since imprecise probabilities can be interpreted in terms of buying prices. However, these "buying prices" are not the same as Walley's ones [4], which are concerned with bets and rewards about the result of an experiment. In our case there are not necessarily bets and rewards.

It does not mean that our proposal cannot be used to interpret fuzzy measures that are actually measures of uncertainty. For instance, let μ be an additive fuzzy measure. In this case, μ is clearly a measure of uncertainty (more precisely, a probability). Now, we shall see how we can interpret it within our context:

Example 5 *Consider a random experiment and its corresponding sample space Ω. Assume we have a certain amount of money available to bet about the events in 2^Ω. Let us consider that amount of money to be equal to 1. Then, this quantity can be regarded as the total amount of resources available to the realization of the experiment.*

We may try to model our uncertainty about the result of the experiment by means of a measure μ such that for each $A \subseteq \Omega$, $\mu(A)$ represents the fraction of money we are willing to bet to receive one unit if A occurs. It can be interpreted, within our context, as the fraction of resources consumed if A occurs.

However, in the general case we have not been able to define a framework for interpreting fuzzy measures as measures of uncertainty. Though we do not have results to support it, we think that this framework could consist of situations where the elements in Ω are ordered.

3 Partial Information and Coherence

As we pointed out before, in many situations it can be difficult to get a complete specification of the measure. For instance, in the very simple situation in example 4, we would need to specify 14 values. This number grows exponentially in the size of Ω.

However, it can be feasible to obtain the measure for some subsets of Ω. In this case we say that we have a *partial information* over Ω. The formal definition is as follows:

Definition 1 (Partial information) Let Ω be a finite set of categories. A *partial information* over Ω is a pair (X, σ), where X is a proper subset of 2^{Ω} and σ is a mapping $\sigma : X \to [0, 1]$.

The following definition imposes a restriction to make a partial information be coherent with the interpretation of a fuzzy measure. This notion of coherence is a consequence of the discussion in section 2.

Definition 2 (Coherent partial information) We say that a partial information (X, σ) over Ω is *coherent* if and only if:

1. For every $A, B \in X$ such that $A \subseteq B$, it holds that $\sigma(A) \leq \sigma(B)$.
2. If $\Omega \in X$, then $\sigma(\Omega) = 1$.
3. If $\emptyset \in X$, then $\sigma(\emptyset) = 0$.

In the transportation vehicle example, the concept of coherence means that the fraction of resources consumed if two compartments are occupied may not be lower than if just one of them is occupied.

Example 6 *Consider again the transportation vehicle case. The following is a coherent partial information over* $\Omega = \{c, v, m, b\}$*:*

$$X = \{\{c\}, \{b\}, \{c, v\}, \{c, v, b\}\} \ ,$$
$$\sigma(\{c\}) = 0.3, \ \sigma(\{b\}) = 0.1 \ ,$$
$$\sigma(\{c, v\}) = 0.6, \ \sigma(\{c, v, b\}) = 0.7 \ .$$

4 Extension of a Partial Information

Once we have characterized the coherence of a partial information, it would be desirable to define a rule to make inferences about the measure in the sets for which no information is available, that inference being compatible with the partial information and with the operational interpretation. The key point here is the concept of *compatibility*, that we formally define in this way:

Definition 3 (Compatible fuzzy measure) We say that a fuzzy measure μ over Ω is *compatible* with a coherent partial information (X, σ), if for every $A \in X$, $\mu(A) = \sigma(A)$.

It is clear that many fuzzy measures can be compatible with a given coherent partial information.

The concept of compatibility allows to make inferences about the measure of the sets that are not elements of X. This inference should produce, for each set not in X, an interval where any measure compatible with (X, σ) must lie. To achieve this, we define the next two measures:

Definition 4 (Lower compatible measure) Let (X, σ) be a coherent partial information. We define the *lower compatible measure* with respect to (X, σ) as

$$\mu_*(A) = \begin{cases} \max\limits_{\substack{B \in X \\ B \subseteq A}} \{\sigma(B)\}, & \text{if } \exists B \in X \text{ s.t. } B \subseteq A \ , \\ \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

for all $A \subseteq \Omega$.

Definition 5 (Upper compatible measure) Let (X, σ) be a coherent partial information. We define the *upper compatible measure* with respect to (X, σ) as

$$\mu^*(A) = \begin{cases} \min\limits_{\substack{B \in X \\ A \subseteq B}} \{\sigma(B)\}, & \text{if } \exists B \in X \text{ s.t. } A \subseteq B \ , \\ \\ 1 & \text{otherwise.} \end{cases} \tag{2}$$

for all $A \subseteq \Omega$.

Observe that if $A \in X$, then $\mu_*(A) = \mu^*(A) = \sigma(A)$.

With this, we can define the concept of *extension* of a coherent partial information, that will produce the minimum interval for each set where the measure will lie, with the only restriction of coherence. In other words, extension is intended to be the maximum inference we can make from a coherent partial information with the only restriction of coherence. This concept is analogous to the natural extension of lower previsions [3, 4].

Definition 6 (Extension) Given a coherent partial information (X, σ), we define its *extension* as the pair of measures (μ_*, μ^*), where μ_* and μ^* are as defined above.

Example 7 *Consider the coherent partial information in example 6. Applying extension for making inference about, say* $\{c, b\}$*, would produce the interval* $[0.3, 0.7]$*. It means that every measure* μ *compatible with* (X, σ) *must verify* $0.3 \leq \mu(\{c, b\}) \leq 0.7$*.*

Proposition 1 *Given a fuzzy measure* μ *compatible with a coherent partial information* (X, σ)*, then for every* $A \in 2^{\Omega}$*,* $\mu_*(A) \leq \mu(A) \leq \mu^*(A)$*.*

Proof. We shall distinguish two cases:

- If $A \in X$, by definition of compatible measure, we have that $\mu_*(A) = \sigma(A) = \mu(A)$.

- If $A \notin X$ we have two possibilities:

  a) If $\exists B \in X$ such as $B \subseteq A$, by definition of fuzzy measure $\mu(B) \leq \mu(A)\ \forall B \subseteq A$. By definition of lower compatible measure, $\mu_*(A) = \max\{\sigma(B) \mid B \subseteq A\}$, which is equal to $\max\{\mu(B) \mid B \subseteq A\}$ since μ is compatible with (X, σ). Thus, $\mu_*(A) \leq \mu(A)$.

  b) If $\forall B \subseteq A$, $B \notin X$, $\mu_*(A) = 0 \leq \mu(A)$.

The proof is analogous for upper compatible measures.

Some interesting cases of fuzzy measures compatible with a coherent partial information are measures based on *averaging operators.*

An averaging operator [6] is a function with the following properties:

- Idempotency: $T(x, x) = x$.
- Monotonicity: If $x \leq x'$ and $y \leq y'$ then $T(x, y) \leq T(x', y')$.
- Commutativity: $T(x, y) = T(y, x)$.

Proposition 2 *Let (X, σ) be a compatible partial information over Ω, and (μ_*, μ^*) its extension. Let μ be a mapping over Ω, defined as*

$$\mu(A) = T(\mu_*(A), \mu^*(A)) \quad A \subseteq \Omega \ , \tag{3}$$

with T an averaging operator. Then μ is a fuzzy measure compatible with (X, σ)

Proof. First we prove that μ is a fuzzy measure.

By idempotency, $\mu(\emptyset) = T(\mu_*(\emptyset),\ \mu^*(\emptyset)) = T(0, 0) = 0$ and
$\mu(\Omega) = T(\mu_*(\Omega), \mu^*(\Omega)) = T(1, 1) = 1$.

If $B \subseteq A$, clearly $\mu_*(B) \leq \mu_*(A)$ and $\mu^*(B) \leq \mu^*(A)$. This, together with monotonicity of the averaging operator, implies that $\mu(B) = T(\mu_*(B), \mu^*(B)) \leq T(\mu_*(A), \mu^*(A)) = \mu(A)$. Thus, μ is a fuzzy measure.

Besides, since T is idempotent, for all $A \in X$, $\mu(A) = T(\mu_*(A), \mu^*(A)) = T(\sigma(A), \sigma(A)) = \sigma(A)$. Thus, μ is compatible with (X, σ).

As a consequence, this kind of operators can be used to obtain fuzzy measures compatible with a partial information.

5 An Algorithm for Computing the Extension

In this section we present an algorithm for computing the extension for any given set $A \subseteq \Omega$. For a more efficient arrangement of the computations, we shall make use of the lattice representation of 2^Ω. Figure 1 displays the lattice representation corresponding to the transportation vehicle example.

First of all, we must fix some notation. For any $A \subseteq \Omega$, we shall denote by $\Pi(A)$ the set of direct predecessors of A in the lattice, and by $\Lambda(A)$ the set of direct successors of A in the lattice, considering that Ω is the top and $\emptyset$ the bottom. For instance, it can be checked in Fig.1 that $\Pi(\{c, v\}) = \{\{c, v, m\}, \{c, v, b\}\}$ and $\Lambda(\{c, v\}) = \{\{c\}, \{v\}\}$.

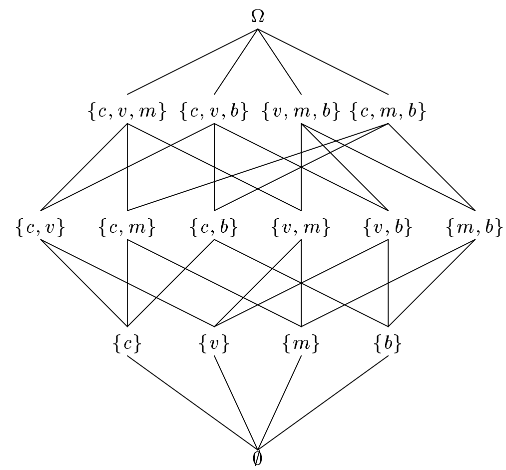

Figure 1: Lattice representation of the power set of $\Omega = \{c, v, m, b\}$.

Now assume we want to compute, for instance, $\mu^*(A)$ for $A \subseteq \Omega$. It could be done by asking to each set B in $\Pi(A)$ for its value $\mu^*(B)$ and then take $\mu^*(A) = \min\{\mu^*(B) \mid B \in \Pi(A)\}$. Analogously, to compute $\mu_*(A)$ it would be enough to know $\mu_*(B)$ for every $B \in \Lambda(A)$ and then taking $\mu_*(A) = \max\{\mu_*(B) \mid B \in \Lambda(A)\}$. This facts allow the specification of a single algorithm to compute the extension of a set $A \subseteq \Omega$ based on two recursive procedures. More precisely, the algorithm can be written as follows, where A is a subset of Ω and (X, σ) a coherent partial information over Ω:

EXTENSION(A,Ω,X,σ**)**

$\pi_*(A) =$**LOWER(**A,Ω,X,σ**)**;

$\pi^*(A) =$**UPPER(**A,Ω,X,σ**)**;

Give $[\pi_*(A), \pi^*(A)]$ as the extension of A.

In the algorithm above, **LOWER** and **UPPER** are the next two recursive procedures:

LOWER(A,Ω,X,σ)

if $A = \emptyset$ **then return** 0;

else

  if $A \in X$ **then return** $\sigma(A)$;

  else

    $M := 0$;

    for each B in $\Lambda(A)$ **do**

      $N :=$**LOWER**(B,Ω,X,σ);

      if $N > M$ **then** $M := N$;

    return M;

which returns the value of the lower compatible measure of A, and

UPPER(A,Ω,X,σ)

if $A = \Omega$ **then return** 0;

else

  if $A \in X$ **then return** $\sigma(A)$;

  else

    $M := 1$;

    for each B in $\Pi(A)$ **do**

      $N :=$**UPPER**(B,Ω,X,σ);

      if $N < M$ **then** $M := N$;

    return M;

which returns the upper compatible measure of A.

Note that this algorithm is designed to be used only if we are interested in the extension for just one set. For instance, assume we want to compute the upper and lower bounds provided by the extension for a set A, and that its successors and predecessors are defined in the partial information. Then, the extension for A is obtained in just one recursion step. This algorithm can be described without recursive procedures; for instance, the lower compatible measure for a set A would be obtained by exploring all the elements in X that are subsets of A and taking the maximum value of σ for them. We have adopted the recursive option for the sake of simplicity in the exposition.

If we are interested in the extension for all the subsets of Ω, a more efficient algorithm can be designed, consisting of two traversals over the lattice structure: one from the top downwards (to compute the upper compatible measure) and one from the bottom upwards (to compute the lower compatible measure). However, this case is less interesting, since we are trying to avoid exponential complexity.

6 Experimental Evaluation

In this section we present the results of an experimental evaluation of the algorithm. The aim of this experimentation is to show the amplitudes of the intervals produced by the extension algorithm for some coherent partial informations.

We have considered four experiments: the first one with 7 elements in Ω and the other ones with 8, 9 and 10 respectively. In each experiment, we have generated 500 coherent partial informations with $|X| = 0.1 \times |2^\Omega|$ (i.e. the cardinal of X being a 10 percent of the cardinal of 2^Ω), 500 with $|X| = 0.2 \times |2^\Omega|$, 500 with $|X| = 0.3 \times |2^\Omega|$ and 500 with $|X| = 0.4 \times |2^\Omega|$. For each partial information, we have computed its extension and the average amplitude of the intervals produced. The goal of the experiment is to check whether we can avoid exponential complexity by specifying the measure just for a reduced number of sets.

The results of the experiments are displayed in figures 2 and 3.

We can see in figure 2 how the average amplitude quickly decreases as the percentage of sets for which some information is provided grows. Also, the average amplitude decreases as the number of elements in Ω increases (see figure 3) for a fixed percentage of sets in X, but not so quickly as in the previous case.

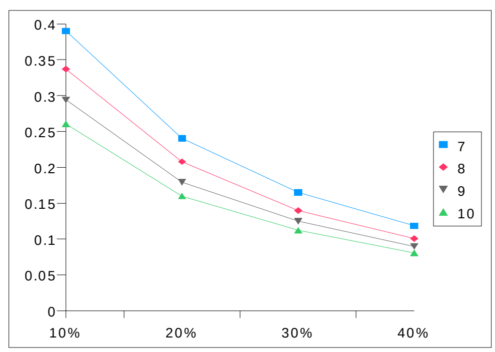

Figure 2: Average amplitude vs. percentage of subsets in X.

7 Conclusions

In this paper we have proposed an operational interpretation of general fuzzy measures. The aim is to give a clear meaning to the numbers, being this mean-

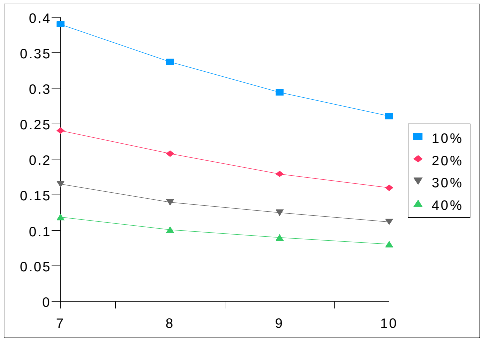

Figure 3: Average amplitude vs. $|\Omega|$.

ing likely to be understood by someone different to the one who assesses the values of the measure. We think that this interpretation can avoid misunderstandings that are quite frequent in the use of fuzzy measures.

On the basis of this interpretation, a concept of coherence can be defined. By coherence we understand the minimum restriction that one must impose to every partial information in such a way that it does not violate the interpretation we formulate. This comes up to match with the concept of monotonicity of a fuzzy measure.

About the extension of a partial information, it is the maximum inference we can do based only in the restriction of coherence. In that sense, it is similar to the concept of natural extension [3].

Many more concepts are to be studied in further works. For instance, how the combination of some measures can be performed under the restriction of coherence. Also, a deeper study is necessary to check the appropriateness of fuzzy measures as measures of uncertainty.

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